1.10 The Stokes-Oseen formula [10] for dragon a sphere at low velocity V is:

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

where D = sphere diameter, μ = viscosity, and ρ = density. Is the formula homogeneous?

Solution: Write this formula in dimensional form, using Table 1-2:

$$\{F\} = \{3\pi\}\{\mu\}\{D\}\{V\} + \left\{\frac{9\pi}{16}\right\}\{\rho\}\{V\}^2\{D\}^2$$

$$\text{or: } \left\{\frac{ML}{T^2}\right\} = \{1\}\left\{\frac{M}{LT}\right\}\{L\}\left\{\frac{L}{T}\right\} + \{1\}\left\{\frac{M}{L^3}\right\}\left\{\frac{L^2}{T^2}\right\}\{L\}^2$$

where, hoping for homogeneity, we have assumed that all constants ($3,\pi$,9,16) are pure, i.e., {unity}. Well, yes indeed, all terms have dimensions $\{ML/T^2\}$! Therefore the Stokes-Oseen formula (derived in fact from a theory) is dimensionally homogeneous

1.22* According to the theory of Chap. 8, as a uniform stream approaches a cylinder of radius R along the line AB shown in Fig. P1.22, $-\infty < x < -R$, the velocities are

$$u = U_\infty(1 - R^2/x^2); \quad v = w = 0$$

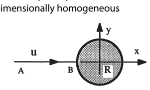


Fig. P1.22

Using the concepts from Ex. 1.5, find (a) the maximum flow deceleration along AB; and (b) its location.

Solution: We see that u slows down monotonically from U_∞ at A to zero at point B, x = –R, which is a flow "stagnation point." From Example 1.5, the acceleration (du/dt) is

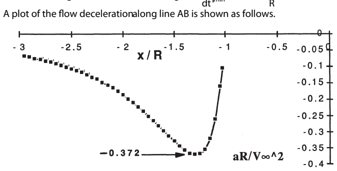
$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 + U_\infty\left(1 - \frac{R^2}{x^2}\right)\left[U_\infty\left(+\frac{2R^2}{x^3}\right)\right] = \frac{U_\infty^2}{R}\left(\frac{2}{\zeta^3} - \frac{2}{\zeta^5}\right), \quad \zeta = \frac{x}{R}$$

This acceleration is negative, as expected, and reaches a minimum near point B, which is found by differentiating the acceleration with respect to x:

$$\frac{d}{dx}\left(\frac{du}{dt}\right) = 0 \quad \text{if } \zeta^2 = \frac{5}{3}, \quad \text{or} \quad \frac{x}{R}\Big|_{\max\,decel.} \approx -1.291 \quad \text{Ans. (b)}$$

$$\text{Substituting } \zeta = -1.291 \text{ into (du/dt) gives } \frac{du}{dt}\Big|_{\min} = -0.372\frac{U_\infty^2}{R} \quad \text{Ans. (a)}$$

A plot of the flow deceleration along line AB is shown as follows.



1.26 A tire has a volume of 3.0 ft^3 and a 'gage' pressure of 32 psi at 75°F. If the ambient pressure is sea-level standard, what is the weight of air in the tire?

Solution: Convert the temperature from 75°F to 535°R. Convert the pressure to psf:

$$p = (32\ lbf/in^2)(144\ in^2/ft^2) + 2116\ lbf/ft^2 = 4608 + 2116 \approx 6724\ lbf/ft^2$$

From this compute the density of the air in the tire:

$$\rho_{air} = \frac{p}{RT} = \frac{6724\ lbf/ft^2}{(1717\ ft\cdot lbf/slug\cdot°R)(535\ °R)} = 0.00732\ slug/ft^3$$

Then the total weight of air in the tire is

$$W_{air} = \rho g \upsilon = (0.00732\ slug/ft^3)(32.2\ ft/s^2)(3.0\ ft^3) \approx 0.707\ lbf \quad \text{Ans.}$$

1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

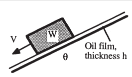


Fig. P1.45

Solution: Let "x" be down the incline, in the direction of V. By "terminal" velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W\sin\theta - \tau A = W\sin\theta - \left(\mu\frac{V}{h}\right)A = ma_x = 0,$$

$$\text{or: } V_{terminal} = \frac{hW\sin\theta}{\mu A} \quad \text{Ans}$$

1.65 The system in Fig. P1.65 is used to estimate the pressure p1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C. Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water; and (b) mercury.

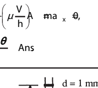


Fig. P1.65

Solution: This is a somewhat more realistic variation of Ex. 1.9. Use values from that example for contact angle θ:

(a) Water at 60°C: $\gamma \approx 9640\ N/m^3$, $\theta \approx 0°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.0662\ N/m)\cos(0°)}{(9640\ N/m^3)(0.001\ m)} = 0.0275\ m,$$

$$\text{or: } \Delta h_{true} = 15.0 - 2.75\ cm \approx 12.25\ cm\ (+22\%\ error) \quad \text{Ans. (a)}$$

(b) Mercury at 60°C: $\gamma \approx 132200\ N/m^3$, $\theta \approx 130°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.47\ N/m)\cos 130°}{(132200\ N/m^3)(0.001\ m)} = -0.0091\ m,$$

$$\text{or: } \Delta h_{true} = 15.0 + 0.91 \approx 15.91\ cm\ (-6\%\ error) \quad \text{Ans. (b)}$$

1.75 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally-spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

Solution: The absolute maximum length L occurs when the pump inlet pressure is slightly greater than 20 kPa. The pump increases this by 1.3 MPa and friction drops the pressure over a distance L until it again reaches 20 kPa. In other words, quite simply,

$$1.3\ MPa = 1{,}300{,}000\ Pa = (150\ Pa/m)L, \quad \text{or} \quad L_{max} \approx 8660\ m \quad \text{Ans.}$$

It makes more sense to have the pump inlet at 1 atm, not 20 kPa, dropping L to about 8 km.

2.1 For the two-dimensional stress field in Fig. P2.1, let

$$\sigma_{xx} = 3000\ psf \quad \sigma_{yy} = 2000\ psf$$
$$\sigma_{xy} = 500\ psf$$

Find the shear and normal stresses on plane AA cutting through at 30°.

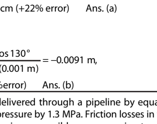
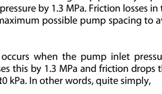


Fig. P2.1

Solution: Make cut "AA" so that it just hits the bottom right corner of the element. This gives the freebody shown at right. Now sum forces normal and tangential to side AA. Denote side length AA as "L."



$$\sum F_{n,AA} = 0 = \sigma_{AA}L - (3000\sin 30 + 500\cos 30)L\sin 30 - (2000\cos 30 + 500\sin 30)L\cos 30$$

$$\text{Solve for} \quad \sigma_{AA} \approx 2683\ lbf/ft^2 \quad \text{Ans. (a)}$$

$$\sum F_{t,AA} = 0 = \tau_{AA}L - (3000\cos 30 - 500\sin 30)L\sin 30 - (500\cos 30 - 2000\sin 30)L\cos 30$$

$$\text{Solve for} \quad \tau_{AA} \approx 683\ lbf/ft^2 \quad \text{Ans. (b)}$$

2.4 Given a flow pattern with isobars $p_o - Bz + Cx^2$ = constant. Find an expression x = fcn(z) for the family of lines everywhere parallel to the local pressure gradient ∇p.

Solution: Find the slope (dx/dz) of the isobars and take the negative inverse and integrate:

$$\frac{d}{dz}(p_o - Bz + Cx^2) = -B + 2Cx\frac{dx}{dz} = 0, \quad \text{or:} \quad \frac{dx}{dz}\Big|_{p=const} = \frac{B}{2Cx} = \frac{-1}{(dx/dz)_{gradient}}$$

$$\text{Thus } \frac{dx}{dz}\Big|_{gradient} = -\frac{2Cx}{B}, \quad \text{integrate } \int\frac{dx}{x} = \int\frac{-2C\,dz}{B}, \quad x = \text{const } e^{-2Cz/B} \quad \text{Ans.}$$

2.8 A diamond mine is 2 miles below sea level. (a) Estimate the air pressure at this depth. (b) If a barometer, accurate to 1 mm of mercury, is carried into this mine, how accurately can it estimate the depth of the mine?

Solution: (a) Convert 2 miles = 3219 m and use a linear-pressure-variation estimate:

$$\text{Then } p \approx p_a + \gamma h = 101{,}350\ Pa + (12\ N/m^3)(3219\ m) = 140{,}000\ Pa \approx 140\ kPa \quad \text{Ans. (a)}$$

Alternately, the troposphere formula, Eq. (2.27), predicts a slightly higher pressure:

$$p \approx p_a(1 - Bz/T_o)^{5.26} = (101.3\ kPa)[1 - (0.0065\ K/m)(-3219\ m)/288.16\ K]^{5.26} = 147\ kPa \quad \text{Ans. (a)}$$

(b) The gage pressure at this depth is approximately 40,000/133,100 ≈ 0.3 m Hg or 300 mm Hg ±1 mm Hg or ±0.3% error. Thus the error in the actual depth is 0.3% of 3220 m or about ±10 m if all other parameters are accurate. Ans. (b)

2.14 The closed tank in Fig. P2.14 is at 20°C. If the pressure at A is 95 kPa absolute, determine p at B (absolute). What percent error do you make by neglecting the specific weight of the air?

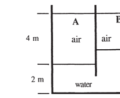


Fig. P2.14

Solution: First compute $\rho_A = p_A/RT$ = (95000)/[287(293)] ≈ 1.13 kg/m^3, hence $\gamma_A \approx$ (1.13)(9.81) ≈ 11.1 N/m^3. Then proceed around hydrostatically from point A to point B:

$$95000\ Pa + (11.1\ N/m^3)(4.0\ m) + 9790(2.0) - 9790(4.0) - \left(\frac{p_B}{RT}\right)(9.81)(2.0) = p_B$$

$$\text{Solve for} \quad p_B \approx 75450\ Pa \quad \text{Accurate answer.}$$

If we neglect the air effects, we get a much simpler relation with comparable accuracy:

$$95000 + 9790(2.0) - 9790(4.0) \approx p_B \approx 75420\ Pa \quad \text{Approximate answer.}$$

2.15 In Fig. P2.15 all fluids are at 20 °C. Gage A reads 15 lbf/in^2 absolute and gage B reads 1.25 lbf/in^2 less than gage C. Compute (a) the specific weight of the oil; and (b) the actual reading of gage C in lbf/in^2 absolute.

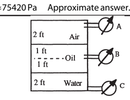


Fig. P2.15

Solution: First evaluate $\gamma_{air} = (p_A/RT)g$ = [15 × 144/(1717 × 528)](32.2) ≈ 0.0767 lbf/ft^3. Take γ_{water} = 62.4 lbf/ft^3. Then apply the hydrostatic formula from point B to point C:

$$p_B + \gamma_{oil}(1.0\ ft) + (62.4)(2.0\ ft) = p_C = p_B + (1.25)(144)\ psf$$

$$\text{Solve for} \quad \gamma_{oil} \approx 55.2\ lbf/ft^3 \quad \text{Ans. (a)}$$

With the oil weight known, we can now apply hydrostatics from point A to point C:

$$p_C = p_A + \sum\rho gh = (15)(144) + (0.0767)(2.0) + (55.2)(2.0) + (62.4)(2.0)$$

$$\text{or: } p_C = 2395\ lbf/ft^2 = 16.6\ psi \quad \text{Ans. (b)}$$

2.20 The hydraulic jack in Fig. P2.20 is filled with oil at 56 lbf/ft^3. Neglecting piston weights, what force F on the handle is required to support the 2000-lbf weight shown?

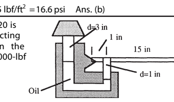


Fig. P2.20

Solution: First sum moments clockwise about the hinge A of the handle:

$$\sum M_A = 0 = F(15 + 1) - P(1),$$

or: F = P/16, where P is the force in the small (1 in) piston.

Meanwhile figure the pressure in the oil from the weight on the large piston:

$$p_{oil} = \frac{W}{A_{3\text{-}in}} = \frac{2000\ lbf}{(\pi/4)(3/12\ ft)^2} = 40744\ psf,$$

$$\text{Hence} \quad P = p_{oil}A_{small} = (40744)\frac{\pi}{4}\left(\frac{1}{12}\right)^2 = 222\ lbf$$

Therefore the handle force required is F = P/16 = 222/16 ≈ 14 lbf Ans.

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than $P_{atmosphere}$?

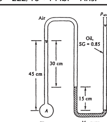


Fig. P2.45

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$p_{atm} + (0.85)(9790)(0.4\ m) - (133100)(0.15\ m) - (12)(0.30\ m) + (9790)(0.45\ m) = p_A,$$

$$\text{or: } p_A = p_{atm} - 12200\ Pa = 12200\ Pa\ (vacuum) \quad \text{Ans.}$$

2.60 The pressure in the air gap is 8000 Pa gage. The tank is cylindrical. Calculate the net hydrostatic force (a) on the bottom of the tank; (b) on the cylindrical sidewall CC; and (c) on the annular plane panel BB.

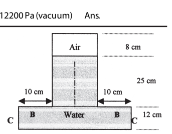


Fig. P2.60

Solution: (a) The bottom force is simply equal to bottom pressure times bottom area:

$$p_{bottom} = p_{air} + \rho_{water}g|\Delta z| = 8000\ Pa + (9790\ N/m^3)(0.25 + 0.12\ m) = 11622\ Pa\text{-gage}$$

$$F_{bottom} = p_{bottom}A_{bottom} = (11622\ Pa)(\pi/4)(0.36\ m)^2 = 1180\ N \quad \text{Ans (a)}$$

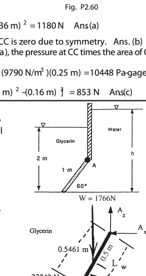
(b) The net force on the cylindrical sidewall CC is zero due to symmetry. Ans. (b)

(c) The force on annular region CC is, like part (a), the pressure at CC times the area of CC:

$$p_{CC} = p_{air} + \rho_{water}g|\Delta z|_{CC} = 8000\ Pa + (9790\ N/m^3)(0.25\ m) = 10448\ Pa\text{-gage}$$

$$F_{CC} = p_{CC}A_{CC} = (10448\ Pa)(\pi/4)[(0.36\ m)^2 - (0.16\ m)^2] = 853\ N \quad \text{Ans (c)}$$

2.61 Gate AB in Fig. P2.61 is a homogeneous mass of 180 kg, 1.2 m wide into the paper, resting on smooth bottom B. All fluids are at 20°C. For what water depth h will the force at point B be zero?



Solution: Let γ = 12360 N/m^3 for glycerin and 9790 N/m^3 for water. The centroid of AB is 0.433 m vertically below A, so h_{CG} = 2.0 – 0.433 = 1.567 m, and we may compute the glycerin force and its line of action:

$$F_g = \gamma h A = (12360)(1.567)(1.2) = 23242\ N$$

$$y_{CP,g} = -\frac{(1/12)(1.2)(1)^3\sin 60°}{(1.567)(1.2)} = -0.0461\ m$$

These are shown on the freebody at right. The water force and its line of action are shown without numbers, because they depend upon the centroidal depth on the water side:

$$F_w = (9790)h_{CG}(1.2)$$

$$y_{CP} = -\frac{(1/12)(1.2)(1)^3\sin 60°}{h_{CG}(1.2)} = -\frac{0.0722}{h_{CG}}$$

The weight of the gate, W = 180(9.81) = 1766 N, acts at the centroid, as shown above. Since the force at B equals zero, we may sum moments counterclockwise about A to find the water depth:

$$\sum M_A = 0 = (23242)(0.5461) + (1766)(0.5\cos 60°) - (9790)h_{CG}(1.2)(0.5 + 0.0722/h_{CG})$$

$$\text{Solve for} \quad h_{CG,water} = 2.09\ m, \quad \text{or:} \quad h = h_{CG} + 0.433 = 2.52\ m \quad \text{Ans.}$$

2.62 Gate AB in Fig. P2.62 is 15 ft long and 8 ft wide into the paper, hinged at B with a stop at A. The gate is 1-in-thick steel, SG = 7.85. Compute the 20°C water level h for which the gate will start to fall.

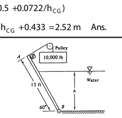
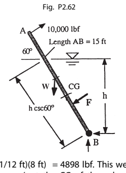


Fig. P2.62

Solution: Only the length (h csc 60°) of the gate lies below the water. Only this part contributes to the hydrostatic force shown in the freebody at right:



$$F = \gamma h_{CG}A = (62.4)\left(\frac{h}{2}\right)(8h\csc 60°) = 288.2h^2\ (lbf)$$

$$y_{CP} = -\frac{(1/12)(8)(h\csc 60°)^3\sin 60°}{(h/2)(8h\csc 60°)} = -\frac{h}{6}\csc 60°$$

The weight of the gate is (7.85)(62.4 lbf/ft^3)(15 ft)(1/12 ft)(8 ft) = 4898 lbf. This weight acts downward at the CG of the full gate as shown (not the CG of the submerged portion). Thus, W is 7.5 ft above point B and has moment arm (7.5 cos 60° ft) about B.

We are now in a position to find h by summing moments about the hinge line B:

$$\sum M_B = (10000)(15) - (288.2h^2)[(h/2)\csc 60° - (h/6)\csc 60°] - 4898(7.5\cos 60°) = 0,$$

$$\text{or: } 110.9h^3 = 150000 - 18369, \quad h = (131631/110.9)^{1/3} = 10.6\ ft \quad \text{Ans.}$$

2.66 Dam ABC in Fig. P2.66 is 30 m wide into the paper and is concrete (SG ≈ 2.40). Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over? Would fluid seepage under the dam change your argument?

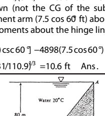
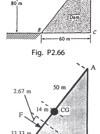


Fig. P2.66

Solution: The centroid of surface AB is 40 m deep, and the total force on AB is

$$F = \gamma h_{CG}A = (9790)(40)(100 \times 30) = 1.175E9\ N$$

The line of action of this force is two-thirds of the way down along AB, or 66.67 m from A. This is seen either by inspection (A is at the surface) or by the usual formula:



$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(30)(100)^3\sin(53.13°)}{(40)(30 \times 100)} = -16.67\ m$$

to be added to the 50-m distance from A to the centroid, or 50 + 16.67 = 66.67 m. As shown in the figure, the line of action of F is 2.67 m to the left of a line up from C normal to AB. The moment of F about C is thus

$$M_C = FL = (1.175E9)(66.67 - 64.0) \approx 3.13E9\ N\cdot m \quad \text{Ans}$$

This moment is counterclockwise, hence it cannot tip over the dam. If there were seepage under the dam, the main support force at the bottom of the dam would shift to the left of point C and might indeed cause the dam to tip over.

3.3 For steady laminar flow through a long tube (see Prob. 1.12), the axial velocity distribution is given by $u = C(R^2 - r^2)$, where R is the tube outer radius and C is a constant. Integrate u(r) to find the total volume flow Q through the tube.

Solution: The area element for this axisymmetric flow is $dA = 2\pi r\,dr$. From Eq. (3.7),

$$Q = \int u\,dA = \int_0^R C(R^2 - r^2)2\pi r\,dr = \frac{\pi}{2}CR^4 \quad \text{Ans.}$$

3.8 Three pipes steadily deliver water at 20°C to a large exit pipe in Fig. P3.8. The velocity V_2 = 5 m/s, and the exit flow rate Q_4 = 120 m^3/h. Find (a) V_1; (b) V_3; and (c) V_4 if it is known that increasing Q_3 by 20% would increase Q_4 by 10%.

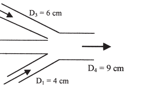


Fig. P3.8

Solution: (a) For steady flow we have $Q_1 + Q_2 + Q_3 = Q_4$, or

$$V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \qquad (1)$$

Since $0.2Q_3 = 0.1Q_4$, and $Q_4 = (120\ m^3/h)(h/3600\ s) = 0.0333\ m^3/s$,

$$V_3 = \frac{Q_4}{2A_3} = \frac{(0.0333\ m^3/s)}{\frac{\pi}{2}(0.06^2)} = 5.89\ m/s \quad \text{Ans. (b)}$$

Substituting into (1),

$$V_1\left(\frac{\pi}{4}\right)(0.04^2) + (5)\left(\frac{\pi}{4}\right)(0.05^2) + (5.89)\left(\frac{\pi}{4}\right)(0.06^2) = 0.0333 \qquad V_1 = 5.45\ m/s \quad \text{Ans(a)}$$

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333\ m^3/s) = V_4(\pi)(0.06^2)/4 \qquad V_4 = 5.24\ m/s \quad \text{Ans. (c)}$$

3.16 An incompressible fluid flows past an impermeable flat plate, as in Fig. P3.16, with a uniform inlet profile $u = U_o$ and a cubic polynomial exit profile

$$u \approx U_o\left(\frac{3\eta - \eta^3}{2}\right) \quad \text{where } \eta = \frac{y}{\delta}$$

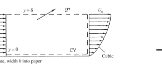


Fig. P3.16

Compute the volume flow Q across the top surface of the control volume.

Solution: For the given control volume and incompressible flow, we obtain

$$0 = Q_{top} + Q_{right} - Q_{left} = Q + \int_0^\delta U_o\left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)b\,dy - \int_0^\delta U_o b\,dy$$

$$= Q + \frac{5}{8}U_o b\delta - U_o b\delta, \quad \text{solve for } Q = \frac{3}{8}U_o b\delta \quad \text{Ans.}$$

3.20 Oil (SG-0.91) enters the thrust bearing at 250 N/hr and exits radially through the narrow clearance between thrust plates. Compute (a) the outlet volume flow in mL/s, and (b) the average outlet velocity in cm/s.

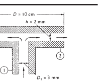


Fig. P3.20

Solution: The specific weight of the oil is (0.91)(9790) = 8909 N/m^3. Then

$$Q_2 = Q_1 = \frac{250/3600\ N/s}{8909\ N/m^3} = 7.8 \times 10^{-6}\ \frac{m^3}{s} = 7.8\ \frac{mL}{s} \quad \text{Ans (a)}$$

$$\text{But also} \quad Q_2 = V_2\pi(0.1\ m)(0.002\ m) = 7.8 \times 10^{-6}, \quad \text{solve for } V_2 = 1.24\ \frac{cm}{s} \quad \text{Ans (b)}$$

3.41 In Fig. P3.41 the vane turns the water jet completely around. Find the maximum jet velocity V_o for a force F_o.

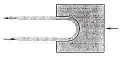


Fig. P3.41

Solution: For a CV enclosing the vane and the inlet and outlet jets,

$$\sum F_x = -F_o = \dot m_{out}u_{out} - \dot m_{in}u_{in} = \dot m_{jet}(-V_o) - \dot m_{jet}(+V_o)$$

$$\text{or: } F_o = 2\rho_o A_o V_o^2, \quad \text{solve for } V_o = \sqrt{\frac{F_o}{2\rho_o(\pi/4)D_o^2}} \quad \text{Ans.}$$

3.44 Consider uniform flow past a cylinder with a V-shaped wake, as shown. Pressures at (1) and (2) are equal. Let b be the width into the paper. Find a formula for the force F on the cylinder due to the flow. Also compute $C_D = F/(\rho U^2 Lb)$.

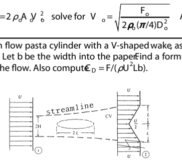


Fig. P3.44

Solution: The proper CV is the entrance (1) and exit (2) plus streamlines above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

$$0 = \int_2 \rho u\,dA - \int_1 \rho u\,dA = 2\int_0^L \rho\frac{U}{2}\left(1 + \frac{y}{L}\right)b\,dy - 2\rho UbH,$$

where 2H is the inlet height. Solve for H = 3L/4.

Now the linear momentum relation is used. Note that the drag force F is to the right (force of the fluid on the body) thus the force F of the body on fluid is to the left. We obtain,

$$\sum F_x = 0 = \int_2 u\rho u\,dA - \int_1 u\rho u\,dA = 2\int_0^L \frac{U}{2}\left(1 + \frac{y}{L}\right)\rho\frac{U}{2}\left(1 + \frac{y}{L}\right)b\,dy - 2H\rho U^2 b = -F_{drag}$$

$$\text{Use} \quad H = \frac{3L}{4}, \quad \text{then } F_{drag} = \frac{3}{2}\rho U^2 Lb - \frac{7}{6}\rho U^2 Lb \approx \frac{1}{3}\rho U^2 Lb \quad \text{Ans}$$

The dimensionless force, or drag coefficient $F/(\rho U^2 Lb)$, equals C_D = 1/3. Ans.

3.54 For the pipe-flow reducing section of Fig. P3.54, D_1 = 8 cm, D_2 = 5 cm, and p_2 = 1 atm. All fluids are at 20°C. If V_1 = 5 m/s and the manometer reading is h = 58 cm, estimate the total horizontal force resisted by the flange bolts.

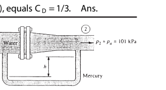


Fig. P3.54

Solution: Let the CV cut through the bolts and through section 2. For the given manometer reading, we may compute the upstream pressure:

$$p_1 - p_2 = (\gamma_{merc} - \gamma_{water})h = (132800 - 9790)(0.58\ m) \approx 71300\ Pa\ (gage)$$

Now apply conservation of mass to determine the exit velocity:

$$Q_1 = Q_2, \quad \text{or} \quad (5\ m/s)(\pi/4)(0.08\ m)^2 = V_2(\pi/4)(0.05)^2, \quad \text{solve for } V_2 \approx 12.8\ m/s$$

Finally, write the balance of horizontal forces:

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \dot m(V_2 - V_1),$$

$$\text{or: } F_{bolts} = (71300)\frac{\pi}{4}(0.08)^2 - (998)\frac{\pi}{4}(0.08)^2(5.0)[12.8 - 5.0] \approx 163\ N \quad \text{Ans}$$

3.61 A 20°C water jet strikes a vane on a tank with frictionless wheels, as shown. The jet turns and falls into the tank without spilling. If θ = 30°, estimate the horizontal force F needed to hold the tank stationary.

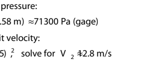


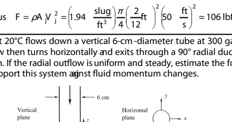
Fig. P3.61

Solution: The CV surrounds the tank and wheels and cuts through the jet, as shown. We have to assume that the splashing into the tank does not increase the x-momentum of the water in the tank. Then we can write the CV horizontal force relation:

$$\sum F_x = -F = \frac{d}{dt}\left(\int u\rho\,d\upsilon\right)_{tank} - \dot m_{in}u_{in} = 0 - \dot m V_{jet} \text{ independent of } \theta$$

$$\text{Thus} \quad F = \rho A_j V_j^2 = \left(1.94\ \frac{slug}{ft^3}\right)\frac{\pi}{4}\left(\frac{2}{12}ft\right)^2\left(50\ \frac{ft}{s}\right)^2 \approx 106\ lbf \quad \text{Ans}$$

3.74 Water at 20°C flows down a vertical 6-cm-diameter tube at 300 gal/min, as in the figure. The flow then turns horizontally and exits through a 90° radial duct segment 1 cm thick, as shown. If the radial outflow is uniform and steady, estimate the forces (F_x, F_y, F_z) required to support this system against fluid momentum changes.



Solution: First convert 300 gal/min = 0.01893 m^3/s, hence the mass flow is ρQ = 18.9 kg/s. The vertical-tube velocity (down) is V_{tube} = 0.01893/[$(\pi/4)(0.06)^2$] = –6.69 k m/s. The exit tube area is $(\pi/2)R\Delta h$ = $(\pi/2)(0.15)(0.01)$ = 0.002356 m^2, hence $V_{exit} = Q/A_{exit}$ = 0.01893/0.002356 = 8.03 m/s. Now estimate the force components:

$$\sum F_x = F_x = \int u_{out}\,d\dot m_{out} = \int_{-45°}^{+45°} -V_{exit}\sin\theta\,\rho\Delta hR\,d\theta\ ?\ 0 \quad \text{Ans (a)}$$

$$\sum F_y = F_y = \int v_{out}\,d\dot m_{out} - \dot m v_{in} = \int_{-45°}^{+45°} -V_{exit}\cos\theta\,\rho\Delta hR\,d\theta - 0 = -V_{exit}\rho\Delta hR\sqrt{2}$$

$$\text{or: } F_y = -(8.03)(998)(0.01)(0.15)\sqrt{2} \approx -17\ N \quad \text{Ans (b)}$$

$$\sum F_z = F_z = \dot m(w_{out} - w_{in}) = (18.9\ kg/s)[0 - (-6.69\ m/s)] \approx +126\ N \quad \text{Ans (c)}$$

3.133 The long pipe in Fig. 3.133 is filled with water at 20°C. When valve A is closed, $p_1 - p_2$ = 75 kPa. When the valve is open and water flows at 500 m^3/h, $p_1 - p_2$ = 160 kPa. What is the friction head loss between 1 and 2, in m, for the flowing condition?

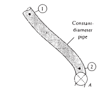


Fig. P3.133

Solution: With the valve closed, there is no velocity or friction loss:

$$\frac{p_1}{\rho g} + z_1 = \frac{p_2}{\rho g} + z_2, \quad \text{or:} \quad z_2 - z_1 = \frac{p_1 - p_2}{\rho g} = \frac{75000}{998(9.81)} \approx 7.66\ m$$

When the valve is open, the velocity is the same at (1) and (2), thus "d" is not needed:

$$\text{With flow: } h_f = \frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) = \frac{160000}{998(9.81)} + 0 - 7.66 \approx 8.7\ m \quad \text{Ans}$$

3.141 Water at 20°C is pumped at 1500 gal/min from the lower to the upper reservoir, as in Fig. P3.141. Pipe friction losses are approximated by $h_f \approx 27V^2/(2g)$, where V is the average velocity in the pipe. If the pump is 75 percent efficient, what horse-power is needed to drive it?

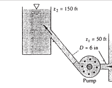


Fig. P3.141

3.145 The large turbine in Fig. P3.145 diverts the river flow under a dam as shown. System friction losses are h_f = $3.5V^2/(2g)$, where V is the average velocity in the supply pipe. For what river flow rate in m^3/s will the power extracted be 25 MW? Which of the two possible solutions has a better "conversion efficiency"?

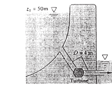


Fig. P3.145

Solution: The flow rate is the unknown, with the turbine power known:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f + h_{turb}, \quad \text{or:} \quad 0 + 0 + 50 = 0 + 0 + 10 + h_f + h_{turb}$$

$$\text{where} \quad h_f = 3.5V_{pipe}^2/(2g) \quad \text{and} \quad h_p = P_p/(\gamma Q) \quad \text{and} \quad V_{pipe} = \frac{Q}{(\pi/4)D_{pipe}^2}$$

Introduce the given numerical data (e.g. D_{pipe} = 4 m, P_{pump} = 25E6 W) and solve:

$$Q^3 - 35410Q + 2.261E6 = 0, \quad \text{with roots } Q = +76.5,\ +137.9,\ \text{and } -214.4\ m^3/s$$

The negative Q is nonsense. The large Q (=137.9) gives large friction loss, $h_f \approx$ 21.5 m. The smaller Q (= 76.5) gives $h_f \approx$ 6.6 m, about right. Select $Q_{river} \approx$ 76.5 m^3/s. Ans.

3.169 Once it has been started by sufficient suction, the siphon in Fig. P3.169 will run continuously as long as reservoir fluid is available. Using Bernoulli's equation with no losses, show (a) that the exit velocity V_2 depends only upon gravity and the distance H and (b) that the lowest (vacuum) pressure occurs at point 3 and depends on the distance L + H.

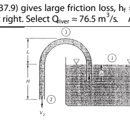


Fig. P3.169

Solution: Write Bernoulli from 1 to 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \approx \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \quad \text{or:} \quad \frac{p_a}{\gamma} + 0 + z_1 \approx \frac{p_a}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\text{Solve for} \quad V_2 = V_{exit} = \sqrt{2g(z_1 - z_2)} = \sqrt{2gH} \quad \text{Ans}$$

Since the velocity is constant throughout the tube, at any point C inside the tube,

$$p_C + \gamma z_C \approx p_a + \gamma z_2, \quad \text{or, at point 3:} \quad p_{C,\min} = p_a - \gamma(z_3 - z_2) = p_a - \rho g(L + H) \quad \text{Ans.}$$

3.172 The 35°C water flow of Fig. P3.172 discharges to sea-level standard atmosphere. Neglecting losses, for what nozzle diameter D will cavitation begin to occur? To avoid cavitation, should you increase or decrease D from this critical value?

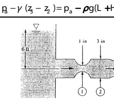


Fig. P3.172

Solution: At 35°C the vapor pressure of water is approximately 5600 Pa (Table A.5). Bernoulli from the surface to point 3 gives the Torricelli result $V_3 = \sqrt{2gh} = \sqrt{2(32.2)(6)} \approx$ 19.66 ft/s. We can ignore section 2 and write Bernoulli from (1) to (3), with $p_1 = p_{vap}$ and Δz = 0:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} \approx \frac{p_2}{\rho} + \frac{V_2^2}{2}, \quad \text{or:} \quad \frac{117}{1.93} + \frac{V_1^2}{2} \approx \frac{2116}{1.93} + \frac{V_3^2}{2},$$

$$\text{but also} \quad V_1 = V_3\left(\frac{D}{1/12}\right)^2$$

$$\text{Eliminate } V_1 \text{ and introduce } V_3 = 19.66\ \frac{ft}{s} \text{ to obtain } D^4 = 3.07E{-4}, \quad D \approx 0.132\ ft \quad \text{Ans}$$

To avoid cavitation, we would keep D < 0.132 ft, which will keep $p_1 > p_{vapor}$.

4.9 An idealized incompressible flow has the proposed three-dimensional velocity distribution

$$\mathbf{V} = 4xy^2\mathbf{i} + f(y)\mathbf{j} - zy^2\mathbf{k}$$

Find the appropriate form of the function f(y) which satisfies the continuity relation.

Solution: Simply substitute the given velocity components into the incompressible continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial}{\partial x}(4xy^2) + \frac{\partial f}{\partial y} + \frac{\partial}{\partial z}(-zy^2) = 4y^2 + \frac{df}{dy} - y^2 = 0$$

$$\text{or: } \frac{df}{dy} = -3y^2. \quad \text{Integrate: } f(y) = \int(-3y^2)\,dy = -y^3 + \text{constant} \quad \text{Ans.}$$

4.19 An incompressible flow field has the cylinder components $\upsilon_\theta = Cr$, $\upsilon_z = K(R^2 - r^2)$, $\upsilon_r = 0$, where C and K are constants and r ≤ R, z ≤ L. Does this flow satisfy continuity? What might it represent physically?

(a) Assuming a no-slip condition at the wall, find an expression for the velocity component v(x, y) for y ≤ δ. (b) Then find the maximum value of v at the station x = 1 m, for the particular case of airflow, when U = 3 m/s and δ = 1.1 cm.

Solution: The two-dimensional incompressible continuity equation yields

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -U\left(\frac{-2y}{\delta^2}\frac{d\delta}{dx} + \frac{2y^2}{\delta^3}\frac{d\delta}{dx}\right), \quad \text{or:} \quad v = 2U\frac{d\delta}{dx}\int_0^y\left(\frac{y}{\delta^2} - \frac{y^2}{\delta^3}\right)dy\Big|_{x=const}$$

$$\text{or: } v = 2U\frac{d\delta}{dx}\left(\frac{y^2}{2\delta^2} - \frac{y^3}{3\delta^3}\right), \quad \text{where } \frac{d\delta}{dx} = \frac{C}{2\sqrt{x}} = \frac{\delta}{2x} \quad \text{Ans. (a)}$$

(b) We see that v increases monotonically with y, thus v_{max} occurs at y = δ:

$$v_{max} = v|_{y=\delta} = \frac{U\delta}{6x} = \frac{(3\ m/s)(0.011\ m)}{6(1\ m)} = 0.0055\ \frac{m}{s} \quad \text{Ans. (b)}$$

This estimate is within 4% of the exact v_{max} computed from boundary layer theory.

4.21 Air flows under steady, approximately one-dimensional conditions through the conical nozzle in Fig. P4.21. If the speed of sound is approximately 340 m/s, what is the minimum nozzle-diameter ratio D_e/D_o for which we can safely neglect compressibility effects if V_o = (a) 10 m/s and (b) 30 m/s?

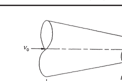


Fig. P4.21

Solution: If we apply one-dimensional continuity to this duct,

$$\rho_o V_o\frac{\pi}{4}D_o^2 = \rho_e V_e\frac{\pi}{4}D_e^2 \quad \text{or} \quad V_o = V_e(D_e/D_o)^2 \text{ if } \rho_o \approx \rho_e$$

To avoid compressibility corrections, we require (Eq. 4.18) that Ma ≤ 0.3 or, in this case, the highest velocity (at the exit) should be V_e ≤ 0.3(340) = 102 m/s. Then we compute

$$(D_e/D_o)_{min} = (V_o/V_e)^{1/2} = (V_o/102)^{1/2} = 0.31 \text{ if } V_o = 10\ m/s \quad \text{Ans (a)}$$
$$= 0.54 \text{ if } V_o = 30\ m/s \quad \text{Ans (b)}$$

4.31 According to potential theory (Chap. 8) for the flow approaching a rounded two-dimensional body, as in Fig. P4.31, the velocity approaching the stagnation point is given by $u = U(1 - a^2/x^2)$, where a is the nose radius and U is the velocity far upstream. Compute the value and position of the maximum viscous normal stress along this streamline. Is this also the position of maximum fluid deceleration? Evaluate the maximum viscous normal stress if the fluid is SAE 30 oil at 20°C, with U = 2 m/s and a = 6 cm.

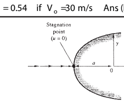


Fig. P4.31

Solution: (a) Along this line of symmetry the convective deceleration is one-dimensional:

$$a_x = u\frac{\partial u}{\partial x} = U\left(1 - \frac{a^2}{x^2}\right)U\left(\frac{2a^2}{x^3}\right) = 2U^2\left(\frac{a^2}{x^3} - \frac{a^4}{x^5}\right)$$

$$\text{This has a maximum deceleration at } \frac{da_x}{dx} = 0, \quad \text{or at } x = -\sqrt{(5/3)}\,a = -1.29a \quad \text{Ans. (a)}$$

The value of maximum deceleration at this point is $a_{x,max} = -0.372U^2/a$.

(b) The viscous normal stress along this line is given by

$$\tau_{xx} = 2\mu\frac{\partial u}{\partial x} = 2\mu\left(\frac{2a^2U}{x^3}\right) \text{ with a maximum } \tau_{max} = \frac{4\mu U}{a} \text{ at } x = -a \quad \text{Ans. (b)}$$

Thus maximum stress does not occur at the same position as maximum deceleration. For SAE 30 oil at 20°C, we obtain the numerical result

$$\text{SAE 30 oil}, \quad \rho = 917\ \frac{kg}{m^3}, \quad \mu = 0.29\ \frac{kg}{m\cdot s}, \quad \tau_{max} = \frac{4(0.29)(2.0)}{(0.06\ m)} \approx 39\ Pa \quad \text{Ans. (b)}$$

4.50 Investigate the polar-coordinate stream function $\psi = Kr^{1/2}\sin\frac{1}{2}\theta$, K = constant. Plot the streamlines in the full xy plane, find any stagnation points, and interpret.

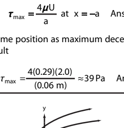


Fig. P4.50

Solution: Simply set ψ/K = constant and plot r versus θ. This represents inviscid flow around a 180° turn. [See Fig. 8.14(e) of the text.]